FINAL REPORT ON THE ONR GRANT NO. N0014-90-J-1705

for May 1990 - January 1994

**"Spectral Analysis, Estimation and Prediction of Multiple Harmonizable Random Fields and Time Series"**



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The following pages present a summary of the work accomplished on the recently concluded ONR Grant No. N00014-90-J-1705, by the P.I. and his students as well as the visitors during the indicated period.



A number of aspects of the original proposal have been solved, although they inspired further work to be considered. This is reflected in a publication of seven (refereed) papers, some of which were also issued as technical reports, and a doctoral dissertation of one student, and two other students who are progressing along towards completion of their thesis work, all under the direction of the P.I. Some of these have received partial support under the grant. The P.I. has spent a sabbatical year during the 1991-'92, in residence, at the Institute for Advanced Study, in Princeton, N.J. and at MSRI in Berkeley, CA, partially supported by the ONR grant during this period. Details

of these activities follows.

1. Research Papers and Reports.

(i) The concept of isotropy is well-known and effectively used for stationary process. In his book on 'Spectral Theory of Random Fields', M. I. Yadrenko shows that there is no nonconstant isotropic stationary random field that is harmonic in the sense that it satisfies the Laplace's equation. This necessitates a suitable generalization of the concept of isotropy for nonstationary classes. Such a result and several properties with applications are included in the paper by the P.I.:

"Sampling and predicti n of harmonizable isotropic random fields," J. Combinatorics, Information and Systems Science, 16 (1991), 207-220.

Using this concept, R. J. Swift has thereafter shown in his thesis that here exist plenty of nonconstant isotropic harmonizable random fields.

A useful boundedness principle was introduced, at the end of his interesting paper in the 3rd Berkeley Symposium on Probability and Mathematical Statistics

(1957), by S. Bochner, pointing out its importance in stochastic analysis but leaving its development for a later time. That aspect was not pursued by the Probabilists for a long time. Recently the P.I. has extended the principle and applied it to filtering and stochastic integrations, unifying the later subject. This work formed the content of the ensuing three papers:

(ii) "$L^{2,2}$ - boundedness, harmonizability, and filtering", Stochastic Analysis and Applications, 10 (1992), 323-342. This paper also contains proofs of certain assertions indicated briefly in Bochner's paper noted above.

(iii) "Stochastic integration: a unified approach," C.R. Acad. Sci (Paris), ser. A., 314 (1992), 629-633. This paper gives a generalization of Bochner's boundedness principle to unify the Itô-type and the Cramér-Kolmogorov-type stochastic integrals. This is the first time such a unification was accomplished.

(iv) A detailed account with multidimensional extension and applications of the preceding results are presented in the general paper:

"An approach to stochastic integration," in Multivariate Analysis: Future Directions, Elsevier Science Publishers (1993), 347-374.

(v) There is a nontrivial problem of evaluation of conditional expectations E(X/Y=a) where the conditioning event has probability zero. In the literature there are suggestions of using a L'Hôpital's rule to evaluate the desired expectation. Unfortunately this procedure leads to multiple answers and paradoxes. Some methods of exact evaluations for the above classes of expectations are given in the paper:

"Exact evaluation of conditional expectations in the Kolmogorov model," Indian J. Math., 35 (1993), 57-70.

(vi) For most of the theoretical work on various classes of stochastic processes, it is necessary to prove their existence which is a key step before one gets into applications. A quite general result for this purpose, to be employed in many problems, is obtained jointly by the P.I. and V. V. Sazonov, in the paper:

"A projective limit theorem for probability spaces and applications," Theor. Prob. Appl. 38 (1993), 345-355.

[This is published in Russian and its English version, by SIAM, will appear soon.]

(vii) Linear unbiased prediction for problems on stochastic flows driven by harmonizable noise is given in a detailed and general paper which includes some inference theory:

"Harmonizable processes and inference: Unbiased prediction for stochastic flows," J. Statist. Plan Inference, 39 (1994), No. 2.

2. Books.

(i) Ever since the classical monographs on a useful class of function spaces, by M. A. Krasnoselskii and Yu. A. Rutickii was published about 35 years ago, most of the open problems listed there have been solved. Moreover, the work itself was generalized and applied in various directions. The P.I. and his students have also made several contributions. Other refinements to the classical case were carried out by some scientists in the P.R.C., including Z. D. Ren and his associates. He came to visit the P.I. at the beginning of 1989, and thereafter a collaborative effort has taken place. A research monograph by the P.I. and Z. D. Ren has been

completed by the latter part of 1991, and the following publication resulted:

"Theory of Orlicz Spaces," Marcel Dekker, Inc., New York (1991), (ix plus 449 pages).

(ii) The P.I. has also been spending considerable portions of his time on various structural and computational problems related to conditional probability measures and their impact in different applications. The material was organized in a sabbatical year from UCR, and finally a research monograph was just completed and published as follows:

"Conditional Measures and Applications," Marcel Dekker, Inc. (1993), (xiv plus 417 pages).

This is the only book of its kind (in any language), completely devoted to conditional probability measures, and should fill in a gap in the literature.

3. Participation in Professional Conferences.

(i) The P.I. has been an invited lecturer at the following institutions in August, 1991, and gave talks there: Two talks in Mathematics Seminars at the Indian

Statistical Institute, in Bangalore, India, and gave three lectures at the Osmania and the Hyderabad Central Universities in Hyderabad.

(ii) The P.I. has attended the AMS meetings in January 1992, in Baltimore, MD, in January 1993, in San Antonio, TX and in January 1994, in Cincinnati, OH. It was useful in participating in some of the sessions each time.

(iii) The P.I. has presented a Member Seminar at the Institute for Advanced Study, Princeton, NJ, in January 1992, and two area seminars at the Mathematical Sciences Research Institute, Berkeley, CA, in May 1992. Additionally, he participated in several other seminars at both these institutions.

(iv) The P.I. is one of the main invited speakers for the International Conference on Multivariate Analysis, held at Penn State University in May 1992. The talk entitled "An approach to stochastic integration," in an expanded version, is later incorporated in the paper in 1(iv) above, which appeared as part of its (refereed) proceedings.

(v) The P.I. was invited to participate in the 3rd International Conference on Function Spaces, held in

Poznan, Poland, in September 1992, as a guest of the Institute of Mathematics of the University there. The invitation was accepted, but could not attend the conference later, due to other work.

4. Graduate Student Training and Support.

During the period of this grant two graduate students have been partially supported. One of the students has completed his Ph.D. dissertation in 1992, and the other has been admitted to candidacy and is working on his thesis. The details are:

(i) Randall J. Swift completed the requirements for his Ph.D. degree in August 1992, entitled: "Structural and Sample Path Analyses of Harmonizable Random Fields." This work is being published in standard journals.

(ii) Another graduate student (Michael L. Green) has been advanced to candidacy and is making satisfactory progress on his dissertation. He received partial support in his studies, and his work is on "multidimensional stochastic integrals" which include continuous harmonizable random fields.

(iii) A third student (H. Soedjak) has also been advanced to candidacy. Although he has not been supported by the

grant for lack of funds, his work is also closely related to that of the grant's proposal. It is on estimation, regression, and asymptotic properties of spectral bimeasure estimators of a class of harmonizable random fields.

The work of both of these students is expected to be completed by the end of this year.

5. Research Scholars Visiting the P.I. at UCR.

(i) Dr. Y. Kakihara of the Tokyo Denki University has been a visiting researcher from April 1990 - July 1991, and again from the middle of 1993 to the present, participating in the P.I.'s seminars and advanced lectures. He is currently working on an extension of the Kalman filter for multidimensional harmonizable random processes.

(ii) Prof. V. V. Sazonov of the Steklov Mathematical Institute has been a visiting scholar for the Fall quarter of 1992, and that resulted in a joint paper by the P.I. and Sazonov, listed as 1(vi) in the research publications above. The continuing ideas in the same area will be taken up when he (hopefully) visits us in the near future.

## 6. Conclusions.

A relatively large part of the grant proposal has been accomplished and the area expanded considerably. This raised new problems of both theoretical and applicational interest. These include the analysis of harmonizable isotropic random fields following R. J. Swift's recent thesis, stochastic integrals for random fields (M. L. Green's dissertation is under preparation), and the asymptotic theory of estimators of spectral bimeasures of harmonizable families (H. Soedjak's work for his thesis), as well as harmonizable currents which has yet to be deeply investigated as it is a promising area with applications in engineering and other areas. The work being done by the graduate students is under the P.I.'s supervision and is expected to be completed within the coming year.

In all the cases, the methods of attack and the mathematical tools needed for completion are significantly different from those used in stationary processes (and fields), even when some of the resulting formulas have recognizable similarities in certain respects. These new problems are expected to be studied in the next and continuing phase of our work, as they contribute both towards mathematics and nontrivial applications.

Reprints of the published work and the technical reports of the

papers under publication are enclosed so that complete details of the above descriptive account is made precise.